RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.Sc. FOURTH SEMESTER TAKE-HOME TEST/ASSIGNMENT, AUGUST 2021

SECOND YEAR [BATCH 2019-22]

Date : 09/08/2021 **MATHEMATICS**

Time : 11am – 1pm **Paper MACT 9** Full Marks : 50

Instructions to the Candidates

- Write your **College Roll No, Year, Subject & Paper Number** on the top of the **Answer Script**.
- Write your **Name, College Roll No, Year, Subject & Paper Number** on the **text box of your e-mail**.
- Read the instructions given at the beginning of each group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be permitted.
- Try to answer all the questions of a single group/unit at the same place.
- All the pages of your answer scripts must be numbered serially by hand.
- In the last page of your answer-scripts, please mention the total number of pages written so that we can verify it with that of the scanned copy of the scripts sent by you.
- For an easy scanning of the answer scripts and also for getting better image, students are advised to write the answers in single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the exam, scan the entire answer script by using Clear Scan: Indy Mobile App OR any other Scanner device and make a **single PDF file (Named as your College Roll No)** and send it to [redacted]

Group A : Partial Differential Equation 1

Answer any three questions from question nos. 1 – 5. All symbols carry their usual meaning. [3 x 5]

1. Obtain the equation of the orthogonal surface to the one parameter family $z = \alpha xy(x^2 + y^2)$, α being the parameter, that passes through the hyperbola $x^2 - y^2 = -\beta^2, z = 0$, where β is a real constant. [5]

2. Find the complete and singular integrals (if they exist) for the pde $2(z + px + qy) = xq^2$. [5]

3. Solve the pde :
$$u_x - 2u_y = 3(x - y) \text{ on the region } \Omega = \{(x, y) \in \mathbb{R}^2 : x > y\}$$
using the method of characteristics, subject to the condition [5]
$$u(x, y) = x \text{ on } \Gamma = \{(x, y) \in \mathbb{R}^2 : x = y\}.$$

4. Obtain a solution for the Laplace equation [5]
$$\triangle\, u = 0 \text{ on } \mathbb{R}^2$$
$$\text{such that } u(x, y) = 0 \text{ on } x = y \text{ line}$$
$$\text{and } u(x, 0) = \frac{x}{2}, \forall x \in \mathbb{R}$$

5. Solve the following pde [5]
$$u_{tt} - u_{xx} = 0 \text{ for } t > 0, x \in (-1, 1)$$
$$\text{with boundary conditions : } u(-1, t) = 0,\ u(1, t) = 0$$
$$\text{and initial conditions : } u(x, o) = \frac{1 + x^3}{2},\ u_t(x, 0) = 0$$

Group B : Laplace Transform

Answer all the questions, maximum one can score 10 in this group.

6. Use convolution theorem to compute: $L^{-1}\left[\frac{s+3}{(s^2+6s+13)^2}\right]$. [4]

7. Use Laplace transform to solve: $y''(t) - y(t) = 51\cos(4t)$; where $y(0) = 0 = y'(0)$. [4]

8. Apply Heaviside expansion theorem to compute: $L^{-1}\left[\frac{21s+27}{s^3-6s^2-27s+140}\right]$. [4]

Group C, Unit I : Basic Number Theory

Answer all the questions, maximum one can score is 20 in this group.

9. (a) If p is prime > 3, prove that $24 \mid (p^2 - 1)$. [3]
   (b) Show that a sum of a pair of twin primes, each greater than 3 is divisible by 12.
   (A pair of twin primes is a pair of primes x and y such that $|x - y| = 2$). [3]

10. (a) Find the missing digit in the number 23104*791 if it is divisible by 9. [2]
    (b) Prove that $43 \mid (6^{n+2} + 7^{2n+1})\ \forall\ n \in \mathbb{N}$. [4]

11. (a) Find the smallest positive integer n such that $\tau(n) = 30$. [4]
    (b) Prove that $\phi(3n) = 3\phi(n)$ iff $3 \mid n$. [4]
    (c) Find the smallest positive integer satisfying the system of linear congruences: [4]

$$\begin{aligned} x &\equiv 2\ (mod\ 2) \\ x &\equiv 3\ (mod\ 3) \\ x &\equiv 1\ (mod\ 5) \end{aligned}$$

Group C, Unit II : Basic Combinatorics

Answer all the questions, maximum one can score is 10 in this group.

12. Find the number of positive integers $x \leq 402$ which are not divisible by any one of 3 and 5. [4]

13. If 52 integers are chosen from the set $\{1, 2, ..., 100\}$, then show that, two can always be found such that the difference of their squares is divisible by 100. [4]

14. Solve the recurrence relation:

$$a_n = -4a_{n-1} - 3a_{n-2} \text{ if } n \geq 2$$

with initial conditions $a_0 = 4$ and $a_1 = 8$. [4]

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